Let’s Do It! 10.1 Null and Alternative Hypotheses

State the null and alternative hypothesis that would be used to test the following statements -- these statements are the researcher’s claim, to be stated as the alternative hypothesis. All hypotheses should be expressed in terms of \( \mu \), the population mean of interest.

(a) The mean age of patients at a hospital is more than 60 years. \( H_0 : \mu = 60 \) versus \( H_1 : \mu > 60 \)

(b) The mean caffeine content in a cup of regular coffee is less than 110 mg. \( H_0 : \mu = 110 \) versus \( H_1 : \mu < 110 \)

(c) The average number of emergency room admissions per day differs from 20. \( H_0 : \mu = 20 \) versus \( H_1 : \mu \neq 20 \)

Let’s Do It! 10.2 Completing a Maze

Experiments on learning in animals sometimes measure how long it takes a mouse to find its way through a maze. The mean time is 18 seconds for one particular maze. A researcher thinks that a loud noise will cause the mice to complete the maze faster. The appropriate null and alternative hypothesis regarding \( m \), the population mean completion time with a noise stimulus, are \( H_0 : \mu = 18 \) versus \( H_1 : \mu < 18 \)

The significance level for this test is set at \( \alpha = .10 \). The researcher decides to measure how long each of 10 mice takes to complete the maze with a noise as stimulus. The sample mean time to complete the race for these 10 mice was 17 seconds. Assume that the 10 mice form a random sample and that the population of completion times with a noise stimulus follows a normal distribution with a known population standard deviation of 2 seconds.

The observed test statistic is given as:

\[
 z = \frac{17 - 18}{2/\sqrt{10}} = -1.58
\]

(a) Sketch the picture to show the \( p \)-value. Label the axis and give the name of the distribution.

(b) Using either the TI or Table II, find the \( p \)-value for the test.

\[
p\text{-value} = P(Z \leq -1.58) = 0.0569 \text{ or } \text{normalcdf(-E99,-1.58)}
\]

(c) Give your decision at a 10% level. Write up your conclusion in words that someone with no background in statistics would understand.

At a significance level of 0.10, we would reject \( H_0 \) because the \( p \)-value of 0.0569 is smaller than the significance level of 0.10. The results were statistically significant at the 10% level. The chance of observing a sample mean completion time of 17 seconds or less would occur approximately 5.7% of the time in many samples, if indeed the noise stimulus had no effect. Thus, based on these data, it appears that the mean completion time under the stimulus of a loud noise is less than 18 seconds.
Let’s Do It! 10.3 Study Time

A new teacher at a university read an article from *The American Freshman* which discussed a study of the amount of time (in hours) college freshmen study each week. The study reported that the mean study time is 7.06 hours. The teacher feels that freshmen at her university study more than 7.06 hr per week on average. The appropriate null and alternative hypotheses in terms of \( \mu \), the population mean number of hours spent studying each week by freshmen at this university, are:

\[
H_0: \mu = 7.06 \quad \text{versus} \quad H_1: \mu > 7.06
\]

The teacher selected a simple random sample of 15 freshmen at her university and found their sample mean study time to be \( \bar{x} = 8.43 \) hours and the sample standard deviation to be \( s = 4.32 \). Assume that study time for freshmen at her university follow a normal distribution.

The observed test statistic is

\[
t = \frac{8.43 - 7.06}{4.32/\sqrt{15}} = 1.23.
\]

(a) Sketch the picture to show the \( p \)-value. Label the axis and give the name of the distribution.

(b) Find the \( p \)-value for the test.

\[
p\text{-value} = P(T \geq 1.23) = 0.1198 \quad \text{or tcdf}(1.23, E99, 14) \text{ or 2:TTest option.}
\]

(c) Are the results statistically significant at the 10% level? **YES**

Explain: Since the \( p \)-value is more than 0.10 we would fail to reject the null hypothesis.

(d) State your conclusion using a well-written sentence.

It appears that the average study time for freshmen is not more than 7.06 hours per week.

Let’s Do It! 10.4 pH Levels

A soil scientist is interested in studying the pH level in the soil for a certain field. In particular, she is going to examine a random sample of soil samples and measure their pH levels to assess if the mean field pH level is neutral, that is, equal to 7, versus the alternative hypothesis that the mean pH level is acidic, that is, less than 7. The significance level will be set at 5%.

(a) State the corresponding null and alternative hypotheses to be tested. \( H_0: \mu = 7 \quad \text{versus} \quad H_1: \mu < 7 \)

Suppose that it is reasonable to assume that the pH levels of all the possible samples that might be drawn are normally distributed. The scientist takes 5 randomly selected samples of soil from the field and measures the pH level in these samples. The pH levels in the sample were: 5.8, 6.3, 6.9, 6.2, and 5.5.

(b) Find the sample mean pH level and the corresponding sample standard deviation.

\[
\bar{x} = 6.14, \quad s = 0.532
\]

(c) Using your TI or Table IV, find the observed test statistic and the corresponding \( p \)-value.

\[
\text{Test Statistic } t = \frac{6.14 - 7}{0.532/\sqrt{5}} = -3.615 \quad \text{where degrees of freedom } = 5-1 = 4, \quad p\text{-value} = P(T \leq -3.61) = 0.0112
\]

(d) The level of significance is 5%. Is the result statistically significant? Explain.

Yes, since the \( p \)-value is less than 0.05. Thus, we would reject the null hypothesis at the 5% level.

(e) State your conclusion using a well written sentence.

We conclude that the average soil pH level is acidic, that is, below 7.
Let’s Do It! 10.5 Market Value for Homes

A real-estate appraiser wants to verify the market value for homes on the east side of the city that are very similar in size and style. The appraiser wants to test the popular belief that the average sales price is $37.80 per square foot for such homes versus that the average differs from $37.80. He will use a significance level of 0.01 and assumes a normal distribution is a good model for sales.

(a) State the appropriate null and alternative hypotheses about \( \mu \), the population mean sales price for such homes.

\[ H_0 : \mu = 37.80 \quad \text{versus} \quad H_1 : \mu \neq 37.80 \]

(b) Suppose that the random sample of six sales were selected. The sampled sales prices per square foot are $35.00, $38.10, $30.30, $37.20, $29.80, and $35.40. Does it appear that the popular perception of the market value is valid for this neighborhood? Give the value of the test statistic and the \( p \)-value.

The test statistic is

\[ t = \frac{34.3 - 37.8}{\frac{3.487}{\sqrt{6}}} = -2.46 \quad \text{and the \( p \)-value} = 2P(T \leq -2.46) = 0.057 \]

(c) Based on your \( p \)-value in part (b), are the results statistically significant at a 1% significance level? Explain.

No, the \( p \)-value is larger than 0.01, so we cannot reject the null hypothesis.

Let’s Do It! 10.6 How Much Beverage?

A beverage dispensing machine is calibrated so that the amount of beverage dispensed is approximately normally distributed with a population standard deviation of 0.15 deciliters (dL).

(a) Compute a 95% confidence interval for the mean amount of beverage dispensed by this machine based on a random sample of 36 drinks dispensing an average of 2.25 deciliters.

\[ 2.25 \pm 1.96 \left( \frac{0.15}{\sqrt{36}} \right) \Rightarrow 2.25 \pm 0.049 \Rightarrow [2.201, 2.299] \quad \text{or using 7:ZInterval with} \ \sigma = 0.15. \]

(b) What is the margin of error for the 95% confidence interval in part (a)? Recall the error margin is the half-width of the confidence interval. The margin of error is 0.049.

(c) From the formula for the margin of error \( E = z^* \left( \frac{\sigma}{\sqrt{n}} \right) \) we can solve for the sample size \( n = \left( \frac{z^* \sigma}{E} \right)^2 \) to have an expression for the required sample size needed for producing an interval with a desired confidence level and a desired margin of error. How large of a sample would you need if you want the margin of error of the 95% confidence interval to be 0.02?

\[ n = \left( \frac{z^* \sigma}{E} \right)^2 = \left( \frac{1.96(0.15)}{0.02} \right)^2 = (14.7)^2 = 216.09 \quad \text{so at least 217. (remember to round up to the next integer)} \]
Let’s Do It! 10.7 Groundhogs
For a random sample of 5 groundhogs from the same geographic region, the mean weight was 15.4 pounds and the standard deviation was 1.25 pounds.
(a) Find an 90% confidence interval for the mean weight of all groundhogs in this geographic region. (show all work)
\[
15.4 \pm 2.132 \left( \frac{1.25}{\sqrt{5}} \right) = 15.4 \pm 1.292 = (14.108, 16.692)
\]
(b) It is stated that the data are a random sample. What other assumption is needed for the confidence interval in part (a) to be valid? (Be Specific) The distribution for the response, weight, must have a normal distribution for the population of all groundhogs from the region.
(c) Is the following statement a correct interpretation of the meaning of the 88% level of confidence? If repeated random samples of 5 groundhogs were obtained, we would expect 88% of the resulting samples to have a sample mean weight that falls in the interval computed in part (a).
Circle: Yes No
Explain: No, we would expect 90% of the resulting confidence intervals to have the population mean weight in it.

Let’s Do It! 10.8 Costs of an Education
How much do full-time students pay for textbooks, on average, for a semester? We wish to produce a 90% confidence interval estimate for the mean cost of textbooks for all full-time students. Take a random sample of 25 full-time students from your class and record how much each spent on textbooks for the semester.
Data Summary Measures sample mean \( \bar{x} = \) __________ sample standard deviation \( s = \) ___________
90% confidence interval Interpretation State the assumptions required and check the assumptions with an appropriate graph of the data. Answers will vary.

Let’s Do It! 10.9 Fireflies Need a Rest Too!
The resting time between flashes of a random sample of 64 fireflies had a mean of 3.77 seconds and a standard deviation of 0.35 seconds. A 90% confidence interval for the mean resting time for this species of firefly is given by (3.70, 3.84). A friend plans to report this information in a paper he is writing. Consider the two summaries:
I. “We can therefore conclude that if this procedure were repeated, the true mean will fall in the interval (3.70, 3.84) 90% of the time.”
II. “We can therefore conclude that if this procedure were repeated, we’d expect 90% of the confidence intervals constructed to contain the true mean.”
Your friend asks you to review his summaries. Your recommendation would be:
I. Use Summary I because this is the correct interpretation and Summary II is too vague.
II. Use Summary II because this is the correct interpretation.
III. Use either summary, since they both are correct interpretations of the results.
If your friend wants to use a significance level of 0.10, does it appear that the mean resting time for this species is 3.90 seconds? State the appropriate hypotheses:

\[ H_0: \mu = 3.9 \quad H_1: \mu \neq 3.9 \]

If your friend wants to use a significance level of 0.10, does it appear that the mean resting time is 3.90 seconds?

Explain. Since this is a two-sided alternative and the significance level is 0.10, we can use the 90% confidence interval to perform the test. The value of 3.9 is not in the 90% confidence interval, so we reject the null hypothesis at the 10% level. The mean resting time appears to be significantly different from 3.9 seconds.

---

**Let’s Do It! 10.10** Infant Sleep Patterns

A study of the sleep patterns of 6-month-old infants in the United States reported a 95% confidence interval for the average amount of time infants sleep (out of every 24 hour period) to be (11.5 hours, 15.2 hours). Suppose we want to test \( H_0: \mu = 15 \) versus \( H_1: \mu \neq 15 \).

At the 5% level, we would: (circle one)  
- Fail to reject \( H_0 \)  
- Reject \( H_0 \)  
- Can’t tell

Explain: The value of 15 falls in the 95% confidence interval, so it is a plausible value for the population mean at the 5% significance level.

At the 1% level, we would: (circle one)  
- Fail to reject \( H_0 \)  
- Reject \( H_0 \)  
- Can’t tell

Explain: Since the 95% confidence interval did contain the value of 15, and a 99% confidence interval would be even wider, the 99% confidence interval will also contain the value of 15.

At the 10% level, we would: (circle one)  
- Fail to reject \( H_0 \)  
- Reject \( H_0 \)  
- Can’t tell

Explain: Even though the 95% confidence interval did contain the value of 15, a 90% confidence interval would be narrower and may or may not contain the value of 15.